Deer Valley Unified

School District

Mathematics Curriculum



Algebra Applications

2010

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**Table of Contents**

Curriculum and Definition Page…………………………………………………… i

Concept Map Definition Page…………………………………………..………… ii

Algebra Applications: An Overview………………………………………….…. iii

Course Considerations………………………………………………….…………. iv

**Topics**

Number and Operations …………………………………………………………… 1

Sequences and Series …………………….……………………………………….. 3

Probability and Statistics……………………..…………………………………… 5

Linear Equations and Inequalities……………………………………..…..……… 7

Systems of Equations and Inequalities………………………..………………..… 9

Functions and Inverse Functions………………………………………..…………. 11

Quadratics and Polynomial Functions……………..………………………………. 13

Exponential Equations and Functions…………………………………………….. 15

Logarithmic, Rational, and Trigonometric Functions……………..……............... 17

Scope and Sequence…………….………………….…..………………………..... 19

**Curriculum Definition Page**

**Topic:**

The organization of performance objectives into a common theme that promotes student engagement and focuses student inquiry.

**Enduring Understanding:**

A central and organizing notion that gives meaning and connection to facts. It has lasting value and can transfer to other inquiries and requires “uncoverage”.

**Standard and Related Concept:**

Identifies the standard and concept from the state standards organized into the topic.

**Performance Objective**

**(2008 Standards):**

Observable and measurable objectives at each grade level from the state standards from 2008.

**E:** The most essential and critical learning.

**I:** Important learning.

**N:** Content that is nice to know but less critical to conceptual understanding and developmental learning.

(Indicated beside the PO in bold)

**Resources:** Materials that would benefit students as they develop a deep understanding of the topic. Includes adopted print and online materials.

**TOPIC:**

**Enduring Understanding:**

**Standard & Related**

**Concept**

**Performance Objectives**

**2008 Standards**

**Resources**

**Performance Objectives**

**2008 Standards**

**EIN**

**Concept Map Definition Page**

**TOPIC:**

**Enduring Understanding:**

**Essential Questions:**

Key Concept

Key Vocabulary

Examples:

**Topic:**

The organization of performance objectives into a common theme that promotes student engagement and focuses student inquiry.

**Essential Question:** The mental questions that help students form a conceptual understanding of the concept or concepts. They point toward key ideas and issues and suggest meaningful and provocative inquiry into content.

**Enduring Understanding:**

A central and organizing notion that gives meaning and connection to facts. It has lasting value and can transfer to other inquiries and requires “uncoverage”.

**Examples:** Examples demonstrate on-target learning and/or help describe how a concept will be taught.

**Key Vocabulary:** The vocabulary that is important for students to know in order to demonstrate an understanding of a topic.

**Key Concept:** The ideas that connect the PO’s to the overarching topic.

# Algebra Applications: An Overview

Algebra Applications is designed to meet a 3rd year math credit for Deer Valley high school students who have historically struggled in high school math courses. Concepts will be introduced, practiced, and assessed using problem solving and applications as the primary method to ensure student growth and success. While the course is intended as a parallel course to Algebra 3-4, essential topics from the standards have been selected for this curriculum so that students will have adequate time to master these power standards. The emphasis of application and problem solving throughout the course will allow students to make connections to their career interests and prepare them with problem solving skills as they transition to post secondary opportunities, including work, community college, or trade school.

This course is an alternative and parallel to Algebra 3-4 and Algebra 3-4 Honors. Students may replace Algebra 3-4 or Algebra 3-4 Honors with Algebra Applications. (Note: No students may take both courses and receive high school credit.) After successful completion of Algebra Applications, students should take College Math and Probability and Statistics to receive a 4th year of math credit. Courses that would be inappropriate for these students, and that they would very likely be unsuccessful in, include College Algebra, Pre-Calculus, AP Statistics, and AP Calculus.

Communication, problem solving, reasoning and proof, connections and representation are the process standards as described in the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM). These process standards are interwoven within all the content strands of the Arizona Mathematics Standard. The process standards emphasize ways to acquire and use the content knowledge.

Mathematics education should enable students to fulfill personal ambitions and career goals in an informational age. In the NCTM *Principles and Standards* document it asks us to “*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it”.[[1]](#footnote-1)* The Arizona Mathematics Standard Articulated by Grade Level is intended to facilitate this vision. (Arizona Mathematics Standard, ADE, 2008)

# Algebra Applications: Course Considerations

* Targeted Students
  + Traditional math instruction has had marginal success
  + Historically struggled in Algebra 1-2 and possibly in Geometry
  + Weak foundational skills from 7th and 8th grade
  + Approaches or Falls Far Below on AIMS as a 10th grade student and possibly from 7th and 8th grade
  + Self perception may be that of a non-math student or “I can’t do math” which leads to lack of motivation
  + Poor attendance history
  + Special Ed students
* Instructional Design
  + Applications and problem solving will be used to introduce and reinforce concepts (Connections is a Math Process Standard)
  + Problem solving is the primary method of instruction ; Put the problem first – then teach the math concepts and skills embedded in the problem (Problem Solving is a Math Process Standard)
  + Chunking topics into smaller, manageable units
  + Pairs and groups allow students to learn collaboratively (Communication is a Math Process Standard)
  + Online support is a tool during instructional time for independent practice
  + Continued review and reinforcement of prior learning, including number concepts
* Assessment Design
  + Formative assessments may be oral and result from the pair and group collaboration during instruction; data from online *Carnegie* reports; in-class white boards
  + Frequent – short to determine the progress of the instruction and the learning in the class and for individuals
  + Problem solving and application should be the focus of each assessment
  + Continued review and reinforcement should be reflected in the assessments
* Grading Considerations
  + 20% of the grade will be determined by a cumulative assessment; the cumulative assessment will consist of 3 parts: 1 – common district multiple choice; 2 – common district written exam; 3 – teacher or school created project/authentic assessment
  + The remaining 80% of the grade will model the Algebra 3-4 grading scheme
  + Multiple opportunities to demonstrate mastery will be provided
* Resources
  + Lap top cart provided through federal funds for the purpose of supporting this course
  + *Carnegie* online and print materials
  + *Mathematics in Action*
  + Graphing calculator are strongly recommended for students but are essential for teachers to have at least for demonstration purposes

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Number and Operations –Essential to Success in the Course – Ongoing Review** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** Integers, rational numbers, irrational numbers and imaginary numbers are used to describe phenomenon in our rapidly changing world. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 1: Number and Operations**  **Concept 1: Number Sense** | HS - PO 1. Explain and demonstrate using examples that to solve certain problems and equations, number systems need to be extended from whole numbers to integers, from integers to rational numbers, and from rational numbers to real numbers. | **E** | throughout |
| **Concept 2: Operations** | HS - PO 1. Solve word problems involving absolute value, powers, roots, and scientific notation. | **I** | *Carmegie* online AA 1 |
| 7th - PO 1. Add, subtract, multiply and divide integers. (students must extend this skill to order of operations, equations, inequalities, matrices) | **E** | *Carmegie* online AA 1 |
| 7th - PO 2. Solve problems with rational numbers and appropriate operations using exact answers or estimates. (students must extend this skill to order of operations, equations, inequalities) | **E** | *Carmegie* online AA 1 |
| HS - PO 2. Summarize the properties of and connections between real number operations; justify manipulations of expressions using the properties of real number operations. | **E** | *Carmegie* online AA 1 |
| HS - PO 3. Calculate powers and roots of rational and irrational numbers. | **E** | *Carmegie* online AA 1 |
| **Concept 3: Estimation** | HS - PO 4. Estimate the location of the rational or irrational numbers on a number line. | **E** | *Carmegie* online AA 1 |
| **Strand 3: Patterns, Algebra and Functions**  **Concept 3: Algebraic Representations** | HS - PO 15. Solve problems using operations with matrices. | **E** | *Carnegie* Alg 2 15.1-2 |
| ***CWR - PO 9. Use matrix operations and the inverse of a matrix to solve problems.*** | **E** | *Carnegie* Alg 2 15.1-2 |

**Key Concepts: Key Vocabulary:**

Absolute Value

TOPIC:

**Number and Operations**

Irrational numbers and rational numbers can be estimated using an understanding of benchmark numbers and the relationship between number systems.

Matrices are an array used to represent phenomenon and they can be added, subtracted and multiplied.

Different kinds of numbers (e.g. radicals, integers, complex) use different rules and procedures to perform basic operations.

Matrix, Matrices

Irrational number

Integers

Rational number

Radical

Simplify:

1. 
2. 
3. 
4. 

Perform matrix operations with integers using



A + B

A – B

A B

Locate the numbers on a number line.

Order the numbers:

Estimate the value:

-6, 3, 

**Essential Question(s):**

* How are integers, rational numbers, irrational numbers and complex numbers used in applications?
* When does an approximation of integers, rational numbers, and irrational numbers or their operations help understand an application or phenomenon in the world?

**Enduring Understanding:** Integers, rational numbers, irrational numbers and imaginary numbers are used to describe phenomenon in our rapidly changing world.

**Examples:**

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Sequences and Series** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** Arithmetic sequences are related to linear functions, and geometric sequences are related to exponential functions. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 1: Patterns** | ***CWR - PO 1. Analyze sequences and series and use them in modeling, including***   * ***explicit formulas for nth terms,*** * ***sums of finite arithmetic series, and*** * ***sums of finite geometric series.*** | **E** | *Carnegie Alg1*  Units 1.2, 1.3, 1.5  Skills Practice 1.1 |
| ***CWR - PO 3. Distinguish between explicit and recursive formulas and convert between them, making good choices about when to use which.*** | **E** | *Carnegie Alg1*  Units 1.2, 1.3, 1.5  Skills Practice 1.1 |
| ***CWR - PO 4. Solve problems involving recursion.*** | **E** | *Carnegie Alg1*  Units 1.2, 1.3, 1.5  Skills Practice 1.1 |
| ***CWR - PO 5. Use and interpret sigma notation to represent summation.*** | **I** | *Carnegie Alg1*  Units 1.2, 1.3, 1.5  Skills Practice 1.1 |

**Key Concepts: Key Vocabulary:**

Arithmetic

The *n*th term, , of an arithmetic sequence with first term and common difference d is given by

+ (n-1) d where n is any positive integer.

TOPIC:

**Sequences and Series**

Generate an explicit formula to calculate the nth term of the geometric sequence…

1. when the first term is 5 and the common ratio is 3.
2. 40, 60, 90, 135, 202.5, …

Suppose your parents give you a $3 allowance after the first week of the year. They agree to increase your allowance by $1 every week for the remainder of the year.

a) What is the total allowance you will have received after the 10th week of the year?

b) What is the total allowance you will have received for the year? (there are 52 weeks in a year!)

**Essential Question(s):**

Why is the *n*th term of a sequence useful?

How can finding the difference (or common ratio) between each term in a sequence help you to find the pattern for the *n*th term?

You want to take swim lessons. There is a one-time pool maintenance fee of $25. And each lesson costs $10. The total cost for different numbers of lessons can be modeled by the sequence:

$35, $45, $55, …

1. How much does it cost to take 5 lessons?
2. How much does it cost to take 15 lessons?
3. Write an algebraic expression for the total cost of *n* lessons.

Find the sum of the arithmetic series:



In a recursive formula, each term is generated from one or more previous terms.

The *n*th term, , of a geometric sequence with first term and common ratio r is given by where n is any positive integer.

**Examples:**

Recursive

Common difference

Common ratio

Explicit formula

Finite sequence

Geometric

induction

infinite sequence

iteration

Sigma notation

**Enduring Understanding:**

A sequence of pictures or numbers can be used to represent problem situations.

The sum of the first n terms of an arithmetic series is given by

The sum of the first n terms of a geometric series is given by

= where

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Probability and Statistics** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** The likelihood of an event happening can be described in terms of probability. Statistical analysis can be used to interpret data. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 2: Data Analysis, Probability, and Discrete Mathematics**  **Concept 1: Data Analysis** | ***PO 2. Compare data sets using graphs and summary statistics, including variance and standard deviation, with or without technology.*** | **E** | *Carnegie* Alg 1  Unit 12.1, 12.4  *Carmegie* online AA 2 |
| **Concept 2: Probability** | ***PO 1. Apply probability concepts to calculate the probability of events and to make informed decisions in practical situations*** | **E** | *Carnegie* Alg 1  Unit 11.2, 11.5  *Carmegie* online AA 2 |
| ***PO 2. Use the principal characteristics of the normal distribution to estimate probabilities.*** | **I** | *Carnegie* Alg 1 |
| ***PO 4. Determine the conditional probability of an event given that another event occurs, decide if two events are dependent or independent, and determine the probability of an event given the probability of the complementary event.*** | **E** | *Carnegie* Alg 1  Unit 11.7  *Carmegie* online AA 2 |
| HS - PO 5. Use concepts and formulas of area to calculate geometric probabilities. | **I** | *Carnegie* Alg 1  Unit 11.8  Skills Practice |
| **Strand 2: Data Analysis, Probability, and Discrete Mathematics**  **Concept 3: Systematic Listing and Counting** | HS - PO 1. Apply the addition and multiplication principles of counting, representing these principles algebraically using factorial notation. | **I** | *Carnegie* Alg 1  Unit 11.5, 11.6 |
| HS - PO 2. Apply appropriate means of computing the number of possible arrangements of items using permutations where order matters, and combinations where order does not matter. | **I** | *Carnegie* Alg 1  Unit 11.5, 11.6 |
| HS - PO 3. Determine the number of possible outcomes of an event. | **I** | *Carnegie* Alg 1  Unit 11.5, 11.6 |

**Key Concepts: Key Vocabulary:**

Biased

TOPIC:

**Probability and Statistics**

There are 6 teams left in the soccer tournament. You think you can predict which teams will get 1st and 2nd place. Your friend says he can predict the top 3 finishers. Which prediction is more likely to be correct?



1. Find the mean for the red team.
2. Find the standard deviation for the red team.

You and your best friend are picking plastic colored eggs from a basket. A total of 10 eggs are in the basket. Only 3 are filled with gift card prizes.

1. You choose an egg first. What is the probability that you will choose one of the prize filled eggs?
2. Suppose you chose a prize filled egg. What is the probability that your friend will also choose a prize filled egg?
3. What the probability that you and your friend will each choose a prize filled egg?

Perform an experiment by flipping a coin 20 times. Then find the theoretical and experimental probability of a coin landing on tails. Compare the two results.

**Examples:**

Unbiased

Theoretical vs experimental

Normal distribution

Sample

Standard deviation

Probability

Measures of central tendency

Conditional probability

Independent vs dependent

Correlation

**Essential Question(s):**

What is a normal distribution?

What is the difference between a biased sample and an unbiased sample?

What is conditional probability?

The probability of an event given that another event has already occurred is the conditional probability.

The graph of a normal distribution is a symmetric, bell shaped curve.

Standard deviation is the square root of the variance represented by *a*.

A sample is unbiased if it is random or unpredictable.

A sample is biased if its design favors certain outcomes.

**Enduring Understanding:**

The likelihood of an event happening can be described in terms of probability.

Statistical analysis can be used to interpret data.

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Linear Equations and Inequalities (including step, absolute value, piecewise)** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 2: Functions and Relationships** | HS - PO 1. Sketch and interpret a graph that models a given context, make connections between the graph and the context, and solve maximum and minimum problems using the graph. | **E** | *Math in Action* Chp 1 |
| HS - PO 2. Determine if a relationship represented by an equation, graph, table, description, or set of ordered pairs is a function. | **E** | *Math in Action* Chp 1.1, 1.3 |
| ***CWR - PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.*** | **E** | *Math in Action* Chp 1 |
| ***CWR - PO 2. Use function notation flexibly: evaluate a function at a value represented by a variable expression*** | **E** | *Math in Action* Chp 1 |
| HS - PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables. | **E** | *Math in Action* Chp 1 |
| ***CWR - PO 7. Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions.*** | **E** | *Math in Action* Chp 1.2 |
| **Concept 3: Algebraic Representations** | ***CWR - PO 1. Rewrite and describe the need for equivalent forms of algebraic expressions.*** | **E** | *Math in Action* Chp 1.8 |
| HS - PO 3. Write an equation given a table of values, two points on the line, the slope and a point on the line, or the graph of the line. | **E** | *Math in Action* Chp 1.6 |
| HS - PO 5. Solve linear equations and equations involving absolute value, with one variable | **E** | *Math in Action* Chp 1.13-15  *Carmegie* online AA 3 |
| HS - PO 6. Solve linear inequalities in one variable. | **E** | *Math in Action* Chp 1.13-15  *Carmegie* online AA 4 |
| **Concept 4: Analysis of Change** | ***CWR - PO 2. Identify patterns in a function’s rate of change, including intervals of increase, decrease, and constancy, and, if possible, relate them to the function’s verbal description or its graph.*** | **E** | *Math in Action* Chp 1.3 |
| **Strand 2: Data Analysis, Probability, and Discrete Math**  **Concept 1: Data Analysis** | ***PO 8. Draw a line of best fit for scatter plot with or without technology; describe how the correlation coefficient relates to fit; and explain when it is appropriate to use the regression equation to make predictions.*** | **E** | *Math in Action* Chp 1.9 |
| **Concept 3: Coordinate Geometry** | HS - PO 5. Graph a linear equation or linear inequality in two variables. | **E** | *Math in Action* Chp 1  *Carmegie* online AA 3, 4 |

**Key Concepts: Key Vocabulary:**

TOPIC:

**Linear Equations and Inequalities**

**(Including step, absolute value, piecewise)**

Absolute value

Domain

Intercept

Range

The length of a screw must be within 0.25 cm of 8 cm.

1. What is the tolerance (or maximum error)?
2. What inequality expresses the acceptable screw lengths?
3. Solve your inequality using a number line, expressing your solution symbolically as a compound inequality.

A line of best fit can be used to predict future events from a scatter plot if there is a correlation in the data.

Inequalities are used when a range of possibilities exists.

Intercepts are where one of the variables is zero. (aka: nothing)

Slope describes the rate of change between the dependent and independent variables.

**Essential Question(s):**

How do you determine when a relation is a function?

What is the difference between an equation and a function?

What is the only type of linear equation that is not a function?

How can the equation for a line be written?

What is the relationship between a linear function, its table and its graph?

Step Functions

Scatter Plot

Piecewise Functions

Line of Best Fit

You are running for class president. You would like to purchase campaign buttons to pass out to students at your school. A company that designs and prints buttons charges a flat fee of $5 plus 75¢ per button for the first 50 buttons purchased. They charge 40¢ per button for the next 50 buttons and then 25¢ for each button purchased after the first 100.

1. Write a function to represent the total cost if you buy *x* buttons.
2. Create a table and a graph from the situation.
3. Use the graph to estimate the number of buttons you can purchase with $125

**Examples:**

You are riding up in a hot air balloon. It has been ascending at a constant rate of 10 ½ feet per minute since the trip started. When you reach a height of 300 feet, you started a log to track your height.

1. Write a function that represents the height of the balloon in terms of minutes since you started the log.
2. Find the height of the balloon after 10 minutes.
3. Create a table from the situation.
4. Graph the projected height of the balloon over time.
5. How long would it take for the balloon to reach a height of 1 mile?

**Enduring Understanding:**

Relationships between dependent and independent variables can be modeled using tables, functions, and graphs.

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Systems of Equations and Inequalities** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** Real world data can be organized into a matrix. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 2: Functions and Relationships** | HS - PO 5. Recognize and solve problems that can be modeled using a system of two equations in two variables. | **E** | *Math in Action* Chp 1.10-12  *Carmegie* online AA 5a, 5B, 5C |
| **Concept 3: Algebraic Representations** | HS - PO 4. Determine from two linear equations whether the lines are parallel, perpendicular, coincident, or intersecting but not perpendicular. | **I** | *Math in Action* Chp 1.7 |
| HS - PO 7. Solve systems of two linear equations in two variables. | **E** | *Math in Action* Chp 1.10-12  *Carmegie* online AA 5a, 5B |
| ***CWR - PO 3. Solve systems of three linear equations in three variables with or without technology.*** | **E** | *Math in Action* Chp 1.12 |
| ***CWR - PO 4. Use matrices to represent everyday problems that involve systems of linear equations.*** | **E** | *Carnegie Alg 2*Chp 15.3 |
| **Strand 4: Geometry and Measurement**  **Concept 4: Coordinate Geometry** | ***CWR - PO 1. Graph the solution set of a system of two or three linear inequalities, and given an ordered pair determine whether it is a solution to the system.*** | **I** | *Math in Action* Chp 1.10-12  *Carmegie* online AA 5C |

**Key Concepts: Key Vocabulary:**

A system of equations has a solution if the graphs of the equations intersect. The solution is the point of intersection.

TOPIC:

**Systems of Equations and Inequalities**

Paula wrote the following matrix equations for the costs of two trips by taxi to Logan Airport in Boston. She use *x* for the cost of round trips and *y* for the cost of one-way trips.



1. Write the equations that are represented by the matrix.
2. What do the coefficients mean in the matrix?
3. What is the cost of a round trip? A one-way trip?
4. Graph the situation.
5. What is the break-even point and what does it represent?

Determinant

System of inequalities

Inconsistent

System of equations

Break-Even Point

Consistent

A car manufacturer is considering two new fuel efficient vehicles to introduce next fall. The production costs for the first model, the EcoRide, will be $275,000 to develop the prototype and $7,500 for each vehicle manufactured. The production costs for the second model, the Green Machine, will be $245,000 to develop the prototype and $10,000 for each vehicle manufactured.

1. What would the cost be to manufacture 100 EcoRides? 100 Green Machines? 500? 1000?
2. Write an equation to represent the cost to manufacture each vehicle.
3. Graph each of them on the same plane.
4. Which vehicle would you recommend the company choose to mass produce? Write a proposal for your recommendations.

**Examples:**

A local TV provider offers three subscription packages.

* Package A: 2 sports channels, 1movie channel, and 1 music channel for $21 per month
* Package B: 3 sports channels, 2 movie channels for $21 per month
* Package C: 1 movie channel, 2 music channels for $19 per month

1. Identify the cost of each channel using the packages available.
2. Another company is offering 4 sports channels, 2 movie channels and 2 music channels for $47 per month. Which company has the “better” offer?
3. Both companies offer a custom package that includes any 10 channels of your choosing. What would be a reasonable price for that package?

Jason is buying wings and hotdogs for a party. One package of wings costs $6.95. Hotdogs cost $3.98 per pound. His budget is at most $40.

1. Write an inequality to represent the cost of the food for the party.
2. Jason knows that he will be buying at least 5 pounds of hotdogs. Write an inequality to represent the situation.
3. Graph both inequalities and identify the portion of the graph that allows Jason to stay within his $40 budget.
4. Identify two of the possible solutions and explain why they will work.
5. Identify a “wings and hotdogs” purchase that will not be feasible for Jason.

A solution to a system of linear inequalities is represented on a graph by shading all the possible points that meet the solution parameters.

Systems of equations (or inequalities) can be solved algebraically using substitution, elimination, or matrices.

**Essential Question(s):**

What kind of situation requires more than one equation as a model?

How can the solution to a system of equations be interpreted?

What is a “break even” point? And how does it apply to a system of equations?

**Enduring Understanding:**

A system of equations allows us to solve problems where there are two or more unknowns.

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Functions and Inverse Functions** | | | Semester this  will be taught: **2** |
| **Enduring Understanding:** Graphing a relationship allows us to model real world situations and visualize the relationship between two variables. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 1: Number and Operations**  **Concept 1: Number Sense** | ***CWR - PO 2. Convert between radical and exponential forms of numerical expressions.*** | **I** | *Math in Action* Chp 2.7  *Carmegie* online AA 6 |
| **Strand 3**: **Patterns, Algebra and Functions**  **Concept 2: Functions and Relationships** | ***CWR - PO 10. Given a function:***   * ***find the inverse of the function,*** * ***determine whether the inverse is a function,*** * ***explain why the graph of a function and its inverse are reflections of each other over the line y = x.*** | **E** | *Math in Action* Chp 2.8  *Carmegie* online AA 6 |
| ***CWR - PO 14. Combine functions by composition, as well as by addition, subtraction, multiplication, and division including any necessary restrictions on the domain.*** | **E** | *Math in Action* Chp 2.1-2.3  *Carmegie* online AA 6 |
| ***CWR - PO 16. Identify the degree of a given polynomial function and write a polynomial function of a given degree.*** | **I** | *Math in Action* Chp 4.11 |
| **Concept 3: Algebraic Representations** | ***CWR - PO 2. Apply the laws of exponents including rational and negative exponents to rewrite expressions in alternative forms.*** | **E** | *Math in Action* Chp 2.4  *Carmegie* online AA 6 |
| ***CWR - PO 6. Divide a polynomial by a lower degree polynomial.*** | **I** | *Carmegie* Alg 2 chp 5.5-5.6 |

**Key Concepts: Key Vocabulary:**

TOPIC:

**Functions and Inverse Functions**

Functions can be added, subtracted, multiplied and divided.

Composition of functions

The inverse of a function has all the same points as the original function, except that the *x*'s and *y*'s have been reversed.

Given: 

Find

1. 
2. 
3. 
4. 
5. 
6. 

The Robertson’s will build a rectangular pool with a stone walkway around it. The total length of the pool and walkway is three times the total width. The walkway is 2 feet wide all around.

1. Write an expression for the area of the pool.
2. Find the area of the pool when the total width is 10 feet.
3. Find the area of the pool when the total width is 9 feet.

In a 30mph collision, according to *Consumer Reports*, the seat belt locks properly 99% of the time. In approximately 90% of such collisions, the air bag will successfully deploy.

1. Write an equation for L(*x*) that represents the number of collisions in which the seat belts lock.
2. Write an equation for D(*x*) that represents the number of collisions in which the air bag deploys.
3. Write an equation for S(*x*) that represents the number of times no one is injured out of *x* collisions.
4. Evaluate S(500)

You are the owner of a small pet kennel. Your kennel can accommodate at most 20 dogs. Your current charge for boarding a dog is $12 per day. Utility bills are approximately $15 per day. The cost of feeding each dog, cleaning its stall, and exercising it is approximately $7.15 per day.

1. Determine an equation that expresses revenue as a function of the number of dogs boarding on a given day.
2. Write an equation that represents the costs for a given day.
3. Use the equations for revenue and cost to determine an equation for the profit (revenue minus costs).
4. Determine the total profit for boarding 12 dogs.

A local newspaper sells advertisement space. The newspaper’s cost to run the ad includes a $12 one time set up fee and an additional 5¢ per letter or character.

1. Write the function that models the cost of running an ad.
2. Write the inverse of this model.
3. Use the inverse function to calculate the number of letters or characters printed in a particular ad in which the newspaper’s total cost to print the ad was $35.

**Examples:**

A function has an inverse if and only if it is one-to-one.

The composite function is obtained by using the output of one function as the input of another.

**Essential Question(s):**

How are a function and its inverse related?

What is the connection between radical expressions and rational exponential expressions?

When does a function have an inverse?

**Enduring Understanding:**

A function expresses idea that one quantity (the input) completely determines another quantity (the output). A function assigns a unique value to each input of a specified type.

Extraneous solutions

Conjugate

Inverse function

Index of a radical

*n*th root

Principal root

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Quadratic and Polynomial Equations and Functions (including radical functions)** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** Graphing a relationship allows us to model real world situations and visualize the relationship between two variables. | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 1: Number and Operations**  **Concept 2: Operations** | ***CWR - PO 1. Explore different forms of complex numbers; determine if the properties of the real number system extend to complex numbers and matrices.*** | **I** | *Math in Action* Chp 4.7  *Carmegie* online AA 8 |
| ***CWR - PO 2. Perform computations with complex numbers.*** | **I** | *Math in Action* Chp 4.7  *Carmegie* online AA 8 |
| ***CWR - PO 3. Describe the relationship between real and complex numbers including plotting complex numbers as points in a plane.*** | **N** | *Math in Action* Chp 4.7  *Carmegie* online AA 8 |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 2: Functions and Relationships** | ***CWR - PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.*** | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| ***CWR - PO 2. Use function notation flexibly: evaluate a function at a value represented by a variable expression.*** | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| HS - PO 4. Use equations, graphs, tables, descriptions, or sets of ordered pairs to express a relationship between two variables. | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| ***CWR - PO 7. Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions.*** | **E** | *Math in Action* Chp 4.2  *Carmegie* online AA 7 |
| HS - PO 6. Recognize and solve problems that can be modeled using a quadratic function. | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| **Concept 3: Algebraic Representations** | ***CWR - PO 5. Simplify radical expressions by performing operations on them.*** | **I** | *Math in Action* Chp 5  *Carmegie* online AA 8 |
| HS - PO 11. Solve square root equations involving only one radical. | **E** | *Math in Action* Chp 5.8-5.9  *Carmegie* online AA 8 |
| HS - PO 12. Factor quadratic polynomials in the form of a*x2 + bx + c* where *a, b,* and *c* are integers. | **E** | *Math in Action* Chp 5 |
| HS - PO 13. Solve quadratic equations. | **E** | *Math in Action* Chp 4.3  *Carmegie* online AA 7 |
| ***CWR - PO 7. Find complex solutions for quadratic equations.*** | **E** | *Math in Action* Chp 5  *Carmegie* online AA 8 |
| ***CWR - PO 8. Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression with and without technology.*** | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| **Concept 4: Analysis of Change** | ***CWR - PO 2. Identify patterns in a function’s rate of change, including intervals of increase, decrease, and constancy, and, if possible, relate them to the function’s verbal description or its graph.*** | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| ***CWR - PO 3. Analyze change in various contexts by modeling and solving word problems using functions and equations.*** | **E** | *Math in Action* Chp 4  *Carmegie* online AA 7 |
| ***CWR - PO 4. Compare relative magnitudes of functions and their rates of change.*** | **E** | *Math in Action* Chp 4 |
| **Strand 4: Geometry and Measurement**  **Concept 2: Transformation of Shapes** | HS - PO 6. Describe how changing the parameters of a linear function affect the shape and position of its graph. | **E** | *Carmegie* online AA 9 |
| ***CWR - PO 1. Describe how changing the parameters of a quadratic function affects the shape and position of its graph f(x) = a(x-h)2+k*** | **E** | *Carmegie* online AA 9 |

**Key Concepts: Key Vocabulary:**

Axis of symmetry

TOPIC:

**Quadratic and Polynomial Equations and Functions**

According to the US Dept. of Transportation, the following table summarizes the average number of gallons of fuel consumed per vehicle from 1960 to 2004.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Year** | 60 | 70 | 80 | 90 | 95 | 2000 | 2002 | 2004 |
| **Gallons** | 668 | 760 | 576 | 520 | 530 | 547 | 555 | 557 |

1. Sketch a scatter plot of the data. Does the data appear to be linear? Explain.
2. Use a graphing calculator to determine and plot the quadratic, cubic, and quartic regression equations for this data. Select the equation that best models the data.
3. What is the practical domain? What is the practical range?
4. Use your equation to estimate the number of gallons of fuel consumed per vehicle in 1955, 1985, and 2008. Which estimate do you have the most confidence in? Explain.

Jason has 80 m of fence to surround an area where he is going to install a dog run. He wants to enclose the largest possible rectangular area.

1. Make a table to represent the possible areas Jason could consider.
2. Write a function that describes the relationship between the area and the width of the dog run.
3. Which width provides the largest possible area? What is that area?
4. Which width results in an area of 0m2?

Identify the basic function and the transformations for each function:

Vertex

Transformations

Rationalize

Radicand

Radical

Parabola

Minimum, maximum

Inverse function

Extraneous solutions

Complex conjugate

Discriminant

Basic function

**Enduring Understanding:** Graphing a relationship allows us to model real world situations and visualize the relationship between two variables.

**Essential Question(s):**

* What transformations can be performed from the most basic form of a function, and how do those transformations affect the graph?
* How are the solutions of an equation related to the graph of a function?
* What is the relationship between the graph, the equation, and the table of values?
* How are the real zeroes of the function related to the solutions of the equation?

A local outlet store charges $2.00 for a four-pack of AA batteries. On an average day, 200 packs are sold. Past sales’ data indicates that the number sold will decrease by 5 packs per day for each 10¢ increase in price.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Selling price | $2 | $2.10 | $2.20 | $2.30 | $2.40 |
| *Number sold* | 200 |  |  |  |  |
| *Revenue* | $400 |  |  |  |  |

1. Complete the table.
2. Write a function that describes the relationship between the revenue and selling price.
3. Graph the function and find the maximum revenue. What selling price provides the maximum revenue?

Simplify:

1. e)
2. f)

**Examples:**

The imaginary unit, *i*, is equal to , and .

Quadratic equations can be solved using factoring, the quadratic formula, completing the square and graphing.

The real numbers are a subset of the complex numbers.

The graph of is a parabola that opens up when a > 0, and opens down when a < 0.

You can find the real zeros of a quadratic function by finding the *x*-intercepts of the graph.

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Exponential Equations and Functions** | | | Semester this  will be taught: **1** |
| **Enduring Understanding:** | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 2: Functions and Relationships** | ***CWR - PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.*** | **E** | *Math in Action* Chp 3.1-3.7  *Carmegie* online AA 10 |
| HS -PO 2. Determine if a relationship represented by an equation, graph, table, description, or set of ordered pairs is a function. | **E** | *Math in Action* Chp 3.1-3.7  *Carmegie* online AA 10 |
| ***CWR - PO 4. Graph exponential functions identifying their key characteristics.*** | **E** | *Math in Action* Chp 3.2-3.7  *Carmegie* online AA 10 |
| ***CWR - PO 5. Sketch the graphs and determine the key characteristics of power functions in the form f(x) = ax^n , a≠ 0, for positive integral values of n.*** | **E** | *Math in Action* Chp 3.2-3.7  *Carmegie* online AA 10 |
| ***CWR - PO 7. Find domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions.*** | **E** | *Math in Action* Chp 3.1-3.7  *Carmegie* online AA 10 |
| **Concept 3: Algebraic Representations** | ***CWR - PO 8. Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression with and without technology.*** | **E** | *Math in Action* Chp 3.1-3.7  *Carmegie* online AA 10 |
| **Concept 4: Analysis of Change** | ***CWR - PO 3. Analyze change in various contexts by modeling and solving word problems using functions and equations.*** | **E** | *Math in Action* Chp 3.1-3.7  *Carmegie* online AA 10 |
| **Strand 4: Geometry and Measurement**  **Concept 2: Transformation of Shapes** | ***CWR - PO 2. Describe how changing the parameters of an exponential function affects the shape and position of its graph (f(x) =abx).*** | **E** | *Carmegie* online AA 11 |

**Key Concepts: Key Vocabulary:**

TOPIC:

**Exponential Equations**

**and Functions**

Asymptote

Base

Change of base formula

Continuous compounding

Decay

Growth

Natural base, e

Compound interest

Your allowance can either be

Plan A: $25 a week or,

Plan B: 1¢ per day the first day, and doubled each successive day.

1. Compare the amount you could earn on Plan A and Plan B after 1 week. Which is more?
2. Compare the amount you could earn on Plan A and Plan B after 2 weeks. 3 weeks? Which is more?
3. Which plan would you prefer and why?
4. Write a function that will calculate the allowance for Plan A for any number of weeks.
5. Write the function that will calculate the allowance for Plan B for any number of days.
6. Compare the two functions.

Chlorine is used to disinfect swimming pools. The chlorine concentration should be approximately 2.5 parts per million (ppm). On sunny, hot days 30% of chlorine dissipates into the air; therefore, the chlorine concentration A(*x*) can be modeled by .

1. What is the initial concentration of chlorine?
2. Create a table for the first 5 days that shows the change in the chlorine concentration if none is added to the pool.
3. Sketch the graph of the chlorine function.
4. The chlorine level should not go below 1.5ppm. When should more chlorine be added to the pool?

**Examples:**

**Essential Question(s):**

* How are exponential functions different from linear, quadratic, and polynomial functions? How are their graphs different?
* What kind of situations are modeled with exponential functions?

**Enduring Understanding:**

Exponential functions increase or decrease multiplicatively.

Graph, state domain and range:



Assume that you have accumulated a credit card balance of $800. The interest charge is calculated using continuous compounding.

1. Write a function to represent the situation described.
2. Find the time it would take for the balance to triple at rate of 6%. At 12%.

“*e*” is the natural base.

*e* is an irrational number approximately equal to 2.71828182845904523536…

If then, *x* = *y*.

Exponential functions can model growth or decay.

|  |  |  |  |
| --- | --- | --- | --- |
| **TOPIC: Logarithmic, Rational and Trigonometric Functions** | | | Semester this  will be taught: **2** |
| **Enduring Understanding:** | | |
| **Standard and**  **Related Concept** | **Performance Objectives** | **EIN** | **Resources**  Ch=Chapter  L=Lesson |
| **Strand 3: Patterns, Algebra, and Functions**  **Concept 2: Functions and Relationships** | ***CWR - PO 1. Express and solve problems that can be modeled using linear, quadratic, logarithmic, exponential, cubic, reciprocal, absolute value, and step and other piecewise-defined functions; interpret their solutions in terms of the context.*** | **E** | *Math in Action* Chp 3.8-3.13; 6.3  *Carmegie* online AA 11 |
| CWR - PO 13. Relate logarithms and exponential functions as inverses; prove basic properties of a logarithm using properties of its inverse; and apply those properties to solve problems. | **I** | *Math in Action* Chp 3.9  *Carmegie* online AA 11 |
| **Concept 3: Algebraic Representation** | HS - PO 8. Simplify and evaluate polynomials, rational expressions, expressions containing absolute value, and radicals. | **E** | *Math in Action* Chp 5.1-5.6 |
| HS - PO 10. Add, subtract, and multiply polynomial and rational expressions. | **E** | *Math in Action* Chp 5.6 |
| **Strand 4: Geometry and Measurement**  **Concept 1: Geometric Properties** | HS - PO 10. Solve problems using right triangles, including special triangles. | **I** | *Math in Action* Chp 6.1, 6.4, 6.5  *Carmegie* online AA 12 |
| HS - PO 11. Solve problems using the sine, cosine, and tangent ratios of the acute angles of a right triangle. | **I** | *Math in Action* Chp 6.1, 6.4, 6.5  *Carmegie* online AA 12 |
| **Concept 2: Transformation of Shapes** | CWR - PO 3. Describe how changing the parameters of a trigonometric function affects the shape and position of its graph (*f(x) = A sin B(x-C)+D* or the other trigonometric functions). | **I** | *Math in Action* Chp 6.3  *Carmegie* online AA 12 |
| CWR - PO 9. Find domain, range, intercepts, period, amplitude, and asymptotes of trigonometric functions. | **I** | *Math in Action* Chp 6.3 |
| **Concept 3: Coordinate Geometry** | CWR - PO 4. Graph all six trigonometric functions identifying their key characteristics. | **I** | *Math in Action* Chp 6.3 *Carmegie* online AA 12 |

**Key Concepts: Key Vocabulary:**

TOPIC:

**Logarithmic, Rational and Trigonometric Functions**

Asymptotes

Find the domain, range, any asymptotes, and graph.

Given: a) 

b) 

Assume the US national debt can be estimated with the model where *x* represents the number of years since 1900 and *y* represents the debt in billions of dollars.

1. According to the model, when did the debt pass $1 trillion?
2. According to the model, what is the annual growth rate of the national debt?
3. What is the doubling time for this growth model?

**Simplify:**

**Solve:** c) 

d) 

The loudness of sound is a function of the listener’s distance from the source of the sound. The relationship between the intensity, I, and the distance, d, can be modeled by the equation , where *k* is a constant. The human voice intensity is modeled when *k* = 1500.

1. Create a table showing the relationship between distance and sound intensity.
2. What is the practical domain?
3. Sketch a graph to model this relationship.
4. As you move closer/farther to the person speaking, what happens to the intensity?
5. What are the asymptotes for this function?
6. Describe the significance for the asymptotes in this problem.

A lighthouse sits on a cliff overlooking the ocean. It is 120 m above the water level. A ship is spotted from the lighthouse at an angle of depression of 52°.

1. Sketch a picture of the situation.
2. How far away is the ship from the edge of the cliff?

**Examples:**

**Essential Question(s):**

* What information do you need to graph a rational function?
* What is an asymptote?
* How are the exponential and logarithmic forms of an expression related?
* What are the properties and behaviors of trig functions?

**Enduring Understanding:**

Logarithms are used to solve for unknown values that are exponents.

Rational functions are not continuous graphs.

All trigonometric values are defined as ratios of sides of right triangles.

Complex fraction

Point discontinuity

Rational function

Reciprocal function

Common logarithms

Variation

To change the base of a log expression use:

Multiplying and dividing rational expressions is similar to multiplying and dividing fractions.

Possible solutions of a rational equation must exclude values that result in zero in the denominator.

To simplify complex fractions, simplify the numerator and the denominator separately, and then simplify the resulting fraction.

Eliminate fractions in rational equations by multiplying each side of the equation by the LCD.

|  |  |
| --- | --- |
| 1st Semester | 2nd Semester |
| **Number and Operations** pp.1-2   * reviewed and reinforced throughout the course * *Carnegie* – Print materials from Alg 2: Chp 15.1, 15.2 * reinforce with optional online *Carnegie* units: * “AA Opt Fractions” * “AA Opt Integers\_PEMDAS”  1. **Sequences and Series** pp. 3-4  * *Carnegie* – Print materials from Alg 1: Chp. 1.2, 1.3, 1.5; * Skills Practice Alg 1: 1.1 * reinforce with online *Carnegie* units: * “AA 1 Rat\_Irrat”  1. **Probability and Statistics** pp. 5-6  * *Carnegie* – Print materials from Alg 1: Chp 11.2, 11.5, 11.6, 11.7, 11.8; Chp 12.1, 12.4 * Skills Practice for Geometric Probability; * reinforce with online *Carnegie* units: * “AA 2 Prob\_Stats”  1. **Linear Equations and Inequalities** pp. 7-8  * Linear Equations and Graphs * *Math in Action* – Chp 1.1-1.9, * reinforce with online *Carnegie*: * AA 3 Linear Models * Linear Inequalities and Absolute Value and Graphs * *Math in Action* – Chp 1.13 – 1.15 * reinforce with online *Carnegie*: * AA 4 Equt\_Inequal  1. **Systems of Equations and Inequalities** pp. 9-10  * *Math in Action* – Chp 1.10-1.12 * *Carnegie* – Print materials from Alg 2: Chp 15.3 * reinforce with online *Carnegie*: * AA 5a Modeling Eq \_Ineq\_Sy * AA 5B Graphing Eq\_Sy * AA5C Var\_Ineq\_Sy | 1. **Functions and Inverse Functions** pp. 11-12  * *Math in Action* – Chp 2.1-2.9 * *Carnegie* – Print materials in Alg 2: Chp 5.5-5.6 * reinforce with online *Carnegie*: * AA 6 Comp\_Inv\_Exp  1. **Quadratic and Polynomial Equations and Functions** pp. 13-14  * Quadratics and Polynomials * *Math in Action* – Chp 4.1-4.6, * reinforce with online *Carnegie*: * AA 7 Quadratics * Complex Numbers and Complex Roots * *Math in Action* – Chp 4.7-4.11 * reinforce with online *Carnegie*: * AA 8 Complex Roots * Radical Functions and Equations * *Math in Action* – Chp 5.7-5.9 * Linear and Quadratic Transformations * *Carnegie* – Print materials from Alg 1: Chp 13.5 * Reinforce with online *Carnegie:* * AA 9 Linear\_Quad Trans  1. **Exponential Equations and Functions** pp. 15-16  * *Math in Action* – Chp 3.1 – 3.7 * Reinforce with online *Carnegie:* * AA 10 Cub\_Expon   **Optional Units:**   1. **Logarithmic, Rational and Trigonometric Functions** pp. 17-18  * Log Functions * *Math in Action* – Chp 3.8 – 3.13 * Reinforce with online *Carnegie:* * AA 11 Log\_Exp\_Transform * Rational Functions * *Math in Action* – Chp 5.1 – 5.6 * Reinforce with online *Carnegie:* * AA 11 Log\_Exp\_Transform * Trigonometry * *Math in Action* – Chp 6.1-6.5 * Reinforce with online *Carnegie:* * AA 12 Trig |

1. National Council of Teachers of Mathematics, Principles and Standardsfor School Mathematics, NCTM Publications, Reston, VA, 2000, p. 3. [↑](#footnote-ref-1)